ELECTROMAGNETIC-FIELD CALCULATION FOR A THERMAL UHF DETECTOR AT HIGH TEMPERATURES. I

A. M. Andrusenko, V. F. Kravchenko,
and V. A. Solodukho

UDC 621.372.8:536.21

Scattering characteristics have been determined from semiinfinite dielectric nonuniformity in a waveguide.

The electrodynamic characteristics of waveguide nonuniformities at UHF are very important in various applications [1, 2]. One can determine the fluctuating electromagnetic field at temperature-type transducers by reference to simpler cases.

1. Symmetrical Nonuniformity in a Rectangular Guide: Formulation

A rectangular guide has a wave propagating from the side $z < 0$, which is of H_{p0} type. This encounters a semiinfinite layered insulator, which results in a system of diffracted waves, whose amplitudes have to be determined. We envisage the case where the insulating plate of dielectric constant ε_1 is placed symmetrically with respect to the axis of the waveguide (Fig. 1a). Then the symmetry of the structure and of the exciting field allow us to divide the problem into the two separate sections, in accordance with the value of p (even or odd). If p is even ($p = 2l, l = 1, 2, 3,\ldots$), an electrical wall placed in the plane $x = 0$ produces no change in the field pattern. The problem therefore reduces to one previously examined [3]. If p is odd ($p = 2l-1, l = 1, 2, 3,\ldots$), the problem is equivalent to that of the structure of Fig. 1b. In that case, a magnetic wall is placed in the plane $x = 0$.

Consider the case of p odd. The solution is sought by means of a modified residue technique, for which purpose we use an auxiliary structure, whose geometry is the same as that of the nonuniformity, but which has an infinitely thin ideally conducting metal strip of width Δ at the boundary (Fig. 1b). If $\Delta \to 0$, we get the initial geometry of the optical obstacle.

In regions A, B, C, and D, we determine the E_y component on the electric field in the form

$$E_y = \begin{cases} A_l \cos\dfrac{\pi(2l-1)}{2a} x \exp(ih_{la}z) + \sum\limits_{m=1}^{\infty} A_{ml} \cos\dfrac{\pi(2m-1)}{2a} x \exp(-ih_{ma}z), & z \leqslant 0, \\ \sum\limits_{m=1}^{\infty} \cos\dfrac{\pi(2m-1)}{2b} x\,[B_{ml}\exp(ih_{mb}z) + \tilde{B}_{ml}\exp(-ih_{mb}z)], & 0 \leqslant z \leqslant \Delta,\ 0 \leqslant x \leqslant b, \\ \sum\limits_{m=1}^{\infty} \sin\dfrac{\pi m}{c}(x-b)\,[C_{ml}\exp(ih_{mc}z) + \tilde{C}_{ml}\exp(-ih_{mc}z)], & 0 \leqslant z \leqslant \Delta,\ b \leqslant x \leqslant a, \\ \sum\limits_{m=1}^{\infty} D_{ml}\Psi_m \exp(ih_{md}z), & z \geqslant \Delta, \end{cases}$$

Khar'kov State Metrology Scientific-Research Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 4, pp. 727-734, October, 1976. Original article submitted August 13, 1975.

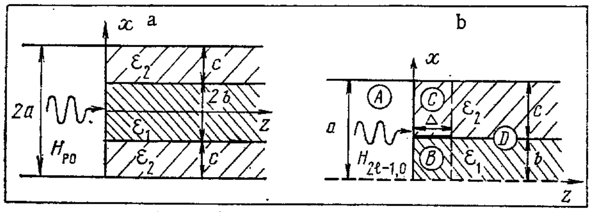

Fig. 1. Semiinfinite laminated dielectric inhomogeneity in a rectangular waveguide: a) test structure; b) structure with additional geometry.

where

$$\Psi_m = \begin{cases} \dfrac{\sin g_{m2} c}{\cos g_{m1} b} \cos g_{m1} x, & 0 \leqslant x \leqslant b, \\ \sin g_{m2}(a - x), & b \leqslant x \leqslant a, \end{cases}$$

$$g_{m1} = \sqrt{k^2 \varepsilon_1 - h_{md}^2}, \quad g_{m2} = \sqrt{k^2 \varepsilon_2 - h_{md}^2},$$

$$h_{ma} = \sqrt{k^2 - \left[\frac{\pi(2m-1)}{2a}\right]^2}, \quad h_{mb} = \sqrt{k^2 \varepsilon_1 - \left[\frac{\pi(2m-1)}{2b}\right]^2},$$

$$h_{mc} = \sqrt{k^2 \varepsilon_2 - \left(\frac{\pi m}{c}\right)^2},$$

and the wave number h_{md} is defined by

$$g_1 \operatorname{tg} g_1 b \operatorname{tg} g_2 c - g_2 = 0.$$

We link up the tangential components of the electromagnetic field at the boundaries z = 0 and z = Δ, eliminate the coefficients B_{ml}, $\tilde{B}_{ml}$, C_{ml}, and $\tilde{C}_{ml}$, eliminate the coefficients, and then put Δ → 0 to get

$$\begin{gathered} \sum_{m=1}^{\infty} \left\{ R_{ml} \left(\frac{1}{h_{ma} + \Gamma_{qb}} + \frac{\rho_{qb}}{h_{ma} - \Gamma_{qb}} \right) + T_{ml} \frac{\tau_{qb}}{h_{md} - h_{qb}} \right\} = A_l \left(\frac{1}{h_{la} - \Gamma_{qb}} + \frac{\rho_{qb}}{h_{la} + \Gamma_{qb}} \right), \\ \sum_{m=1}^{\infty} \left\{ R_{ml} \left(\frac{1}{h_{ma} - \Gamma_{qb}} + \frac{\rho_{qb}}{h_{ma} + \Gamma_{qb}} \right) + T_{ml} \frac{\tau_{qb}}{h_{md} + h_{qb}} \right\} = A_l \left(\frac{1}{h_{la} + \Gamma_{qb}} + \frac{\rho_{qb}}{h_{la} - \Gamma_{qb}} \right), \\ \sum_{m=1}^{\infty} \left\{ R_{ml} \left(\frac{1}{h_{ma} + \Gamma_{qc}} + \frac{\rho_{qc}}{h_{ma} - \Gamma_{qc}} \right) + T_{ml} \frac{\tau_{qc}}{h_{md} - h_{qc}} \right\} = A_l \left(\frac{1}{h_{la} - \Gamma_{qc}} + \frac{\rho_{qc}}{h_{la} + \Gamma_{qc}} \right), \\ \sum_{m=1}^{\infty} \left\{ R_{ml} \left(\frac{1}{h_{ma} - \Gamma_{qc}} + \frac{\rho_{qc}}{h_{ma} + \Gamma_{qc}} \right) + T_{ml} \frac{\tau_{qc}}{h_{md} + h_{qc}} \right\} = A_l \left(\frac{1}{h_{la} + \Gamma_{qc}} + \frac{\rho_{qc}}{h_{la} - \Gamma_{qc}} \right), \\ q = 1, 2, 3, \ldots, \end{gathered} \tag{1}$$

where

$$\begin{gathered} R_{ml} = A_{ml} \cos \frac{\pi(2m-1)}{2a} b, \quad T_{ml} = D_{ml} \sin g_{m2} c, \\ \Gamma_{qb} = \sqrt{k^2 - \left[\frac{\pi(2q-1)}{2b}\right]^2}, \quad \rho_{qb} = \frac{\Gamma_{qb} - h_{qb}}{\Gamma_{qb} + h_{qb}}, \quad \tau_{qb} = \frac{2\Gamma_{qb}}{\Gamma_{qb} + h_{qb}}, \\ \Gamma_{qc} = \sqrt{k^2 - \left[\frac{\pi q}{c}\right]^2}, \quad \rho_{qc} = \frac{\Gamma_{qc} - h_{qc}}{\Gamma_{qc} + h_{qc}}, \quad \tau_{qc} = \frac{2\Gamma_{qc}}{\Gamma_{qc} + h_{qc}}. \end{gathered} \tag{2}$$

2. Symmetrical Nonuniformity in a Cylindrical Waveguide: Formulation

The cylindrical waveguide has a wave propagating from $z < 0$, which is of H_{0p} type, and which encounters an obstacle in the form of semiinfinite piecewise-inhomogeneous dielectric inserts (Fig. 2a). We have to determine the field arising by scattering of the incident wave at this nonuniformity.

The above method is employed. We introduce an auxiliary structure, which is an infinitely thin ideally conducting ring of width Δ and radius b. This ring is coaxial with the waveguide (Fig. 2b). The component E_φ of the electromagnetic field is put as

$$E_\varphi = \begin{cases} A_p^0 J_1\left(\frac{\mu_p}{a} r\right)\exp(ih_{pa}^0 z) + \sum_{m=1}^{\infty} A_{mp}^0 J_1\left(\frac{\mu_m}{a} r\right)\exp(-ih_{ma}^0 z), & z \leqslant 0, \\ \sum_{m=1}^{\infty} J_1\left(\frac{\mu_m}{b} r\right)[B_{mp}^0 \exp(ih_{mb}^0 z) + \tilde{B}_{mp}^0 \exp(-ih_{mb}^0 z)], & 0 \leqslant z \leqslant \Delta,\ 0 \leqslant r \leqslant b \\ \sum_{m=1}^{\infty} Z_c [C_{mp}^0 \exp(ih_{mc}^0 z) + \tilde{C}_{mp}^0 \exp(-ih_{mc}^0 z)], & 0 \leqslant z \leqslant \Delta,\ b \leqslant r \leqslant a, \\ \sum_{m=1}^{\infty} Z_d D_{mp}^0 \exp(ih_{md}^0 z), & z \geqslant \Delta, \end{cases}$$

where

$$h_{ma}^0 = \sqrt{k^2 - \left(\frac{\mu_m}{a}\right)^2}, \quad h_{mb}^0 = \sqrt{k^2\varepsilon_1 - \left(\frac{\mu_m}{b}\right)^2},$$

$$h_{mc}^0 = \sqrt{k^2\varepsilon_2 - \left(\frac{\xi_m}{c}\right)^2},$$

with μ_m the zeros of $dJ_0(x)/dx$, ξ_m the roots of

$$J_1\left(\frac{\xi_m}{c} a\right) N_1\left(\frac{\xi_m}{c} b\right) - J_1\left(\frac{\xi_m}{c} b\right) N_1\left(\frac{\xi_m}{c} a\right) = 0,$$

and γ_m the roots of

$$\nu_m J_0\left(\frac{\nu_m}{c} b\right) F_1 - \gamma_m J_1\left(\frac{\nu_m}{c} b\right) F_0 = 0,$$

$$\nu_m = \sqrt{\gamma_m^2 + k^2 b^2 (\varepsilon_1 - \varepsilon_2)}, \quad h_{md}^0 = \sqrt{k^2\varepsilon_2 - \left(\frac{\gamma_m}{c}\right)^2},$$

$$F_j = J_j\left(\frac{\gamma_m}{c} b\right) - N_j\left(\frac{\gamma_m}{c} b\right) J_1\left(\frac{\gamma_m}{c} a\right) \Big/ N_1\left(\frac{\gamma_m}{c} a\right), \quad j = 0;\ 1,$$

$$Z_c = J_1\left(\frac{\xi_m}{c} r\right) - N_1\left(\frac{\xi_m}{c} r\right) J_1\left(\frac{\xi_m}{c} a\right) \Big/ N_1\left(\frac{\xi_m}{c} a\right),$$

$$Z_d = \begin{cases} J_1\left(\frac{\nu_m}{c} r\right), & 0 \leqslant r \leqslant a, \\ \frac{J_1\left(\frac{\nu_m}{c} b\right)}{F_1}\left[J_1\left(\frac{\gamma_m}{c} r\right) - \frac{J_1\left(\frac{\gamma_m}{c} a\right)}{N_1\left(\frac{\gamma_m}{c} a\right)} N_1\left(\frac{\gamma_m}{c} r\right)\right], & b \leqslant r \leqslant a. \end{cases}$$

We link up the field components at the boundaries of the characteristic irregular regions and proceed as in the first case to get

$$\sum_{m=1}^{\infty}\left\{R_{mp}^0\left(\frac{1}{h_{ma}^0 + \Omega_{qb}} + \frac{\beta_{qb}}{h_{ma}^0 - \Omega_{qb}}\right) + T_{mp}^0 \frac{\sigma_{qb}}{h_{md}^0 - h_{qb}^0}\right\} = A_p^0\left(\frac{1}{h_{pa}^0 - \Omega_{qb}} + \frac{\beta_{qb}}{h_{pa}^0 + \Omega_{qb}}\right),$$

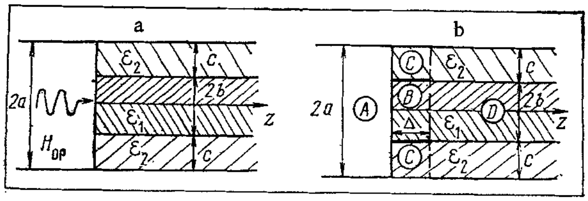

Fig. 2. Semiinfinite piecewise-inhomogeneous dielectric in a cylindrical waveguide: a) test structure; b) structure with additional geometry.

$$\sum_{m=1}^{\infty}\left\{R^0_{mp}\left(\frac{1}{h^0_{ma}-\Omega_{qb}}+\frac{\beta_{qb}}{h^0_{ma}+\Omega_{qb}}\right)+T^0_{mp}\frac{\sigma_{qb}}{h^0_{md}+h^0_{qb}}\right\}=A^0_p\left(\frac{1}{h^0_{pa}+\Omega_{qb}}+\frac{\beta_{qb}}{h^0_{pa}-\Omega_{qb}}\right),$$

$$\sum_{m=1}^{\infty}\left\{R^0_{mp}\left(\frac{1}{h^0_{ma}+\Omega_{qc}}+\frac{\beta_{qc}}{h^0_{ma}-\Omega_{qc}}\right)+T^0_{mp}\frac{\sigma_{qb}}{h^0_{md}-h^0_{qc}}\right\}=A^0_p\left(\frac{1}{h^0_{pa}-\Omega_{qc}}+\frac{\beta_{qc}}{h^0_{pa}+\Omega_{qc}}\right), \quad (3)$$

$$\sum_{m=1}^{\infty}\left\{R^0_{mp}\left(\frac{1}{h^0_{ma}-\Omega_{qc}}+\frac{\beta_{qc}}{h^0_{ma}+\Omega_{qc}}\right)+T^0_{mp}\frac{\sigma_{qc}}{h^0_{md}+h^0_{qc}}\right\}=A^0_p\left(\frac{1}{h^0_{pa}+\Omega_{qc}}+\frac{\beta_{qc}}{h^0_{pa}-\Omega_{qc}}\right),$$

where

$$R^0_{mp}=A^0_{mp}J_1\left(\frac{\mu_m}{a}b\right),\quad T^0_{mp}=D^0_{mp}J_1\left(\frac{\nu_m}{c}b\right),\quad q=1,\ 2,\ 3,\ \ldots,$$

$$\Omega_{qb}=\sqrt{k^2-\left(\frac{\mu_m}{b}\right)^2},\quad \beta_{qb}=\frac{\Omega_{qb}-h^0_{qb}}{\Omega_{qb}+h^0_{qb}},\quad \sigma_{qb}=\frac{2\Omega_{qb}}{\Omega_{qb}+h^0_{qb}}, \quad (4)$$

$$\Omega_{qc}=\sqrt{k^2-\left(\frac{\xi_m}{c}\right)^2},\quad \beta_{qc}=\frac{\Omega_{qc}-h^0_{qc}}{\Omega_{qc}+h^0_{qc}},\quad \sigma_{qc}=\frac{2\Omega_{qc}}{\Omega_{qc}+h^0_{qc}}.$$

3. Solution of the System of Linear Algebraic Equations

Systems (1) and (3) are similar apart from the symbols used, so they can be solved for either case.

The following cases are of particular practical importance: a dielectric step lies at the wall of the waveguide ($\varepsilon_1 = 1$) or the nonuniformity does not touch the wall ($\varepsilon_2 = 1$); we consider one of these. Let $\varepsilon_2 = 1$ (the solution for $\varepsilon_1 = 1$ is analogous). If $\varepsilon_2 = 1$, then ρ_{mc} becomes zero, while $\tau_{mc} = 1$.

Consider the following four integrals over the closed contour C:

$$\frac{1}{2\pi i}\int_C\left\{\frac{f(w)}{w+\Gamma_{qb}}+\rho_{qb}\frac{f(w)}{w-\Gamma_{qb}}+\tau_{qb}\frac{f(w)}{w+h_{qb}}\right\}dw;\quad \frac{1}{2\pi i}\int_C\frac{f(w)}{w+h_{qc}}dw;$$

$$\frac{1}{2\pi i}\int_C\left\{\frac{f(w)}{w-\Gamma_{qb}}+\rho_{qb}\frac{f(w)}{w+\Gamma_{qb}}+\tau_{qb}\frac{f(w)}{w-h_{qb}}\right\}dw;\quad \frac{1}{2\pi i}\int_C\frac{f(w)}{w-h_{qc}}dw; \quad (5)$$

$$q=1,\ 2,\ 3,\ldots,$$

where the function f(w) satisfies the following conditions.

I. f(w) is an analytic function of the complex variable everywhere apart from the points $w = h_{ma}$, $w = -h_{md}$, ($m = 1, 2, 3, \ldots$), and $w = -h_{la}$, where it has simple poles.

II. f(w) has simple zeros at the points $w = \pm h_{qc}$, $q = 1, 2, 3, \ldots$

III. $\operatorname{Res} f(-h_{la}) = A_l$.

IV. f(w) satisfies the following equations for q = 1, 2, 3, ...

$$f(-\Gamma_{qb})+\rho_{qb}f(\Gamma_{qb})+\tau_{qb}\Bigg[f(-h_{qb})-\frac{\operatorname{Res}f(-h_{la})}{h_{la}-h_{qb}}$$

$$+\sum_{s=1}^{\infty}\frac{\operatorname{Res}f(h_{sa})}{h_{sa}+h_{qb}}\Bigg]-\sum_{s=1}^{\infty}\operatorname{Res}f(-h_{sd})\left(\frac{1}{h_{sd}-\Gamma_{qb}}+\frac{\rho_{qb}}{h_{sd}+\Gamma_{qb}}\right)=0.$$

V.

$$f(\Gamma_{qb})+\rho_{qb}f(-\Gamma_{qb})+\tau_{qb}\Bigg[f(h_{qb})-\frac{\operatorname{Res}f(-h_{la})}{h_{la}+h_{qb}}$$

$$+\sum_{s=1}^{\infty}\frac{\operatorname{Res}f(h_{sa})}{h_{sa}-h_{qb}}\Bigg]-\sum_{s=1}^{\infty}\operatorname{Res}f(-h_{sd})\left(\frac{1}{h_{sd}+\Gamma_{qb}}+\frac{\rho_{qb}}{h_{sd}-\Gamma_{qb}}\right)=0.$$

VI. $f(w)=O(w^{-\varkappa})$ for $|w|\to\infty$, where $\varkappa>1$ is determined from the behavior of the field at the edge. For an insulator whose edge is represented by a right angle in the cross section, we have [3]

$$\varkappa=1+\frac{2}{\pi}\arccos\left[\frac{\varepsilon-1}{2(\varepsilon-1)}\right].$$

We reduce the integrals of (5) to sums of residues to get

$$\sum_{m=1}^{\infty}\Bigg\{\operatorname{Res}f(h_{ma})\left(\frac{1}{h_{ma}+\Gamma_{qb}}+\frac{\rho_{qb}}{h_{ma}-\Gamma_{qb}}\right)-\operatorname{Res}f(-h_{md})$$
$$\times\frac{\tau_{qb}}{h_{md}-h_{qb}}\Bigg\}=\operatorname{Res}f(-h_{la})\left(\frac{1}{h_{la}-\Gamma_{qb}}+\frac{\rho_{qb}}{h_{la}+\Gamma_{qb}}\right),$$
$$\sum_{m=1}^{\infty}\Bigg\{\operatorname{Res}f(h_{ma})\left(\frac{1}{h_{ma}-\Gamma_{qb}}+\frac{\rho_{qb}}{h_{ma}+\Gamma_{qb}}\right)-\operatorname{Res}f(-h_{md})$$
$$\times\frac{\tau_{qb}}{h_{md}+h_{qb}}\Bigg\}=\operatorname{Res}f(-h_{la})\left(\frac{1}{h_{la}+\Gamma_{qb}}+\frac{\rho_{qb}}{h_{la}-\Gamma_{qb}}\right),\tag{6}$$
$$\sum_{m=1}^{\infty}\left\{\operatorname{Res}f(h_{ma})\frac{1}{h_{ma}+h_{qc}}-\operatorname{Res}f(-h_{md})\frac{1}{h_{md}-h_{qc}}\right\}=\operatorname{Res}f(-h_{la})\frac{1}{h_{la}-h_{qc}},$$
$$\sum_{m=1}^{\infty}\left\{\operatorname{Res}f(h_{ma})\frac{1}{h_{ma}-h_{qc}}-\operatorname{Res}f(-h_{md})\frac{1}{h_{md}+h_{qc}}\right\}=\operatorname{Res}f(-h_{la})\frac{1}{h_{la}+h_{qc}}.$$

We compare (6) and the simplified system of (1) to get that

$$R_{ml}=\operatorname{Res}f(w)|_{w=h_{ma}},\quad T_{ml}=-\operatorname{Res}f(w)|_{w=-h_{md}}.\tag{7}$$

4. Construction of f(w)

The conditions formulated above define f(w) neatly; the conditions that define the disposition of the zeros and poles of f(w) are met if the function is put in the general form

$$f(w)=R(w)\frac{\prod_{m=1}^{\infty}\left(1-\frac{w}{h_{mc}}\right)\prod_{m=1}^{\infty}\left(1+\frac{w}{h_{mc}}\right)\prod_{m=1}^{\infty}\left(1-\frac{w}{Q_{mb}^{+}}\right)\prod_{m=1}^{\infty}\left(1+\frac{w}{Q_{mb}^{-}}\right)}{\left(1+\frac{w}{h_{la}}\right)\prod_{m=1}^{\infty}\left(1-\frac{w}{h_{ma}}\right)\prod_{m=1}^{\infty}\left(1+\frac{w}{h_{ma}}\right)},\tag{8}$$

where R(w) is an integer function of the variable w that does not equal zero; $\{Q_{mb}^{+}\}$ and $\{Q_{mb}^{-}\}$ are sequences of displaced zeros which have to be determined. However, the f(w) of the form of (8) is not particularly useful here because there are two modified regions in the geometry (auxiliary structure), so the last two products in the numerator in (8) are best replaced by the polynomial

$$P(w)=\left[1+\sum_{m=1}^{\infty}w\left(\frac{U_m}{\Gamma_{mb}-w}+\frac{V_m}{\Gamma_{mb}+w}\right)\right]\prod_{m=1}^{\infty}(1-w/\Gamma_{mb})(1+w/\Gamma_{mb}),\tag{9}$$

which incorporates the perturbation due to the displacement of the zeros from the points $\{\Gamma_{mb}\}$ and $\{-\Gamma_{mb}\}$ and $\{Q^{+}_{mb}\}$ and $\{-Q^{-}_{mb}\}$, respectively.

We substitute for f(w) into the equations that express properties IV and V of f(w) and thus reduce the problem to definition of the sequence of displaced zeros to that of an infinite system of equations for the unknown coefficients U_m and V_m, whose asymptotic behavior for m large can be determined. We put $w = \pm\Gamma_{mb}$ for $m \to \infty$ and use condition VI to get

$$U_m \sim m^{1-\varkappa},\ V_m \sim m^{1-\varkappa},\ m \to \infty.$$

Then (9) for m large becomes as follows:

$$P(w) = \prod_{m=1}^{\infty}\left(1 - \frac{w}{\Gamma_{mb}}\right)\prod_{m=1}^{\infty}\left(1 + \frac{w}{\Gamma_{mb}}\right)\left[1 + \sum_{m=1}^{M_u-1} U_m \frac{w}{\Gamma_{mb} - w}\right.$$
$$\left. + \sum_{m=1}^{M_v-1} V_m \frac{w}{\Gamma_{mb} + w} + \tilde{U}\sum_{m=M_u}^{\infty}\frac{wm^{1-\varkappa}}{\Gamma_{mb} - w} + \tilde{V}\sum_{m=M_v}^{\infty}\frac{wm^{1-\varkappa}}{\Gamma_{mb} + w}\right], \tag{10}$$

where $\tilde{U}$ and $\tilde{V}$ are unknown coefficients; we substitute (10) into (8) and then use conditions IV and V to get a system of $M = M_u + M_v$ linear equations for the unknowns $\{U_m\}$, $\{V_m\}$, $\tilde{U}$, and $\tilde{V}$.

We determine the magnitude of the perturbing coefficients and satisfy the normalization condition III to get the final expression for f(w); this can be used to derive the scattering matrices $\{R_m\}$ and $\{T_m\}$ from (7).

LITERATURE CITED

1. A. M. Andrusenko, V. F. Kravchenko, and V. A. Solodukho, in: Abstracts of the Third All-Union Conference on Electronic Metrology [in Russian], VNIIFTRI (1975), p. 199.
2. L. A. Birger and I. A. Sokolov, Izmeritel'. Tekh., No. 1 (1962).
3. E. G. Royer and R. Mittra, IEEE Trans. Microwave Theory Tech. 20, 273 (1972).